Abstract—This paper presents a force-level planning approach to optimally select and design the orbit of an airborne relay. The objective is to maximize the aggregate performance of communication subscribers served by the airborne relay. The performance is measured as either the duty cycle in terms of signal-to-noise ratio or the throughput achieved by communication subscribers. Both measures are computed with realistic channel, terrain, antenna and aircraft characteristics for ground-to-air and air-to-ground communications. Different methods to generate an airborne relay orbit are discussed with their parameters to optimize. Results are specialized to optimization of elliptical orbit parameters. First, an elliptical orbit is optimally selected with the best center and radius (in X and Y-directions) as orbit parameters subject to the specified altitude and speed of the aircraft. Both exhaustive search and gradient search with reduced complexity are considered to find the best set of orbit parameters. This elliptical orbit is then converted to an operational orbit by optimally selecting orbit waypoints and locating smooth turning points while minimizing the fuel consumption of the airborne relay. A software with interactive GUI is designed and implemented for force-level planning that allows the planner to select different properties of airborne relay and communication subscribers, and returns the optimal orbit for the airborne relay.

Index Terms—Force-level planning; mission planning; airborne relay; orbit; throughput; duty cycle; optimization.

I. INTRODUCTION

Airborne relays (or gateways) are used to provide connectivity, coverage and range extension support to communication subscribers (ground, air, or maritime fixed or mobile users) [1]–[8]. One example of the airborne relay that aims to serve communication subscribers is the Battlefield Airborne Communication Node (BACN) [9]. From the network perspective, the mission success depends on the careful selection of the airborne orbit (trajectory) to provide subscribers with the necessary network communications support.

Network-centric mission planning with airborne relays has two main components: 1) subscriber planning (at the individual node level) and 2) force-level planning (at the network level).

1) Subscriber planning: For a given orbit, the subscriber planning optimizes the performance of each subscriber communicating to/through the dedicated airborne communication platform, such as BACN, and schedules when and at what rate to transmit as the airborne relay moves. This problem has been studied in [1] to maximize the network throughput performance by taking into account various channel, traffic and energy properties.

2) Force-level planning: Given the subscriber planning results computed for all communications subscribers, the force-level planning analyzes the total performance (e.g., connectivity, coverage, or throughput) achieved by communication subscribers for each orbit. This performance is optimized by selecting the best orbit parameters of the airborne relay subject to different orbit constraints and communication requirements.

Subscriber planning has been addressed in [1]. The focus of this paper is the force-level planning. A typical measure for the optimal orbit selection is the coverage (or duty-cycle) such as the percentage of time a communication subscriber is connected with (i.e., it is within the transmission range of) the airborne relay [2], [7]. This measure does not fully consider the underlying communication characteristics. Therefore, we consider maximizing either the duty cycle that reflects the percentage of time the signal-to-noise-ratio (SNR) exceeds a given threshold, or more realistically the actual network throughput, namely the number of packets delivered between communication subscribers and the airborne relay per unit time. Duty cycle accounts for a) predicted channels (calculated based on terrain profile, distance to airborne relay, and antenna characteristics [10]) and b) estimated channels (from pilot signals received from the airborne relay). In addition to those factors, the throughput also accounts for: c) packet traffic, d) (battery) energy properties, and e) interference effects, among other radio-related factors.

The choice for orbit parameters depends on the method used for orbit generation. Examples of orbit parameters include the radius and the center (in X and Y-directions) of elliptical orbits or the waypoints for general orbits. In this paper, we start with discussion on different orbit generation methods and orbit parameters. The force-level planning approach searches for the best set of orbit parameters. Without loss of generality, we focus on the parameter selection for elliptical orbits. First, we optimally select the center and the radius (in semi major and semi minor axis or X and Y-direction) of an elliptical...
orbit. We select these orbit parameters to optimize the duty cycle or the network throughput subject to the specified aircraft properties (altitude and speed). We consider both exhaustive search and gradient search with reduced complexity to solve the optimization problem. Second, we modify this elliptical orbit to an operational orbit by optimally selecting waypoints on this orbit and locating smooth turning points. Third, we optimally select the number of waypoints to minimize the fuel consumption of the airborne relay by limiting the number of turning points. We use real rates of aircraft fuel consumption from [12]. We wrap the orbit generation approach in a force-level planning software that allows the planner to select specifications of airborne relay and communication subscribers, and returns the optimal selection of an airborne relay orbit. We present the design and implementation of the force-level planning software and discuss results.

Our contributions are summarized below:

1) Systematic use of high-fidelity duty cycle and throughput measures (based on realistic channel modeling) for orbit selection of an airborne relay.
2) Optimization framework to select the optimal orbit parameters.
3) Optimization of fuel consumption and turning point selection to establish operational orbits.
4) Software development and performance evaluation.

The rest of the paper is organized as follows. Section II introduces the problem of orbit generation for airborne relays and discusses the sets of orbit parameters to be selected in different orbit generation methods. Section III presents the optimal selection of elliptical orbit parameters through exhaustive and gradient-search approaches. Section IV presents the conversion of an elliptical orbit to an operational orbit while minimizing the fuel consumption of the airborne relay aircraft. Section V discusses the force-level planning software capabilities. Section VI concludes the paper.

II. ORBIT GENERATION

We discuss different ways to generate the orbit for an airborne relay and identify the parameters to be selected.

A. Elliptical Orbit Generation

The $x$ and $y$ coordinates of the elliptical orbit are

$$x(t) = c_x + r_x \cos \left( \frac{2\pi t}{T} \right),$$

$$y(t) = c_y + r_y \sin \left( \frac{2\pi t}{T} \right),$$

where $(c_x, c_y)$ is the center of orbit, $r_x$ and $r_y$ are radius along $x$ and $y$ directions, respectively, and $T$ is the orbit time. Parameters $c_x, c_y, r_x, r_y, T, \text{speed}, s, \text{and altitude}, h$, can be varied to generate different orbits. One sample orbit A (the larger one in red) and one sample orbit B (the smaller one in black) are shown in Fig. 1, where $r_x = 45$ miles, $r_y = 35$ miles, $T = 1908$ seconds, $s = 488$ MPH and $h = 44,000$ feet for orbit A, and $r_x = 22.5$ miles, $r_y = 17.5$ miles, $T = 776$ seconds, $s = 300$ MPH and $h = 44,000$ feet for orbit B. In this paper, we vary parameters $c_x, c_y, r_x$ and $r_y$, and keep others fixed to generate different orbits.

B. Waypoint-based Orbit Generation

Another way to generate an airborne relay orbit is to specify its waypoints. CBAR (Channel Modeling Tool Based on Bidirectional Analytic Ray Tracing and Radiative Transfer) tool [10] can be used to generate an airborne relay orbit. We first add waypoints using CBAR user interface. Based on waypoints, speed and altitude information, CBAR generates the route using AGI’s route design library from Systems Tool Kit [11] integrated within CBAR (illustrated in Fig. 2). CBAR uses real terrain data and real aircraft models in channel computation and route generation. An example of waypoints generated by CBAR is shown in Fig. 3.

Next, we consider the optimal selection of orbit parameters. We start with optimizing the elliptical orbit parameters. Then, we will introduce waypoints to make the orbit operational.
throughput (the number of packets deceived to destination per unit time) sustained by the subscriber. Other metrics can be handled by similar optimization procedures. Consider 15 subscribers (white “×”) and two orbits of the airborne relay as shown in Fig. 4, where orbit 1 (magenta) is generated by an elliptical orbit formula and orbit 2 (yellow) is generated by CBAR using waypoints. As reported in [1], Table I shows the average values of throughput and duty cycle (percentage of time SNR > 1.8dB) for two orbits shown in Fig. 5. Orbit 2 is the better orbit for throughput and orbit 1 is the better orbit for duty cycle.

The next question is how to select the best orbit parameters (either the center or the radius) in a systematic way.

B. Exhaustive Search for Orbit Center

We consider 15 ground users as shown in Fig. 4. These users are located in the area of \([36, 176] \times [-90, 0]\). We search for center within the same area with resolution (grid size) of 4 miles. That is, \(c_x\) can be selected from \(36, 40, ..., 176\) and \(c_y\) can be selected from \(-90, -86, ..., 2\). The search space has \(36 \times 24 = 864\) points. The overall performance can be expressed as a function \(P(c_x, c_y)\), which is determined by subscriber planning such as the one described in [1].

The first approach is the exhaustive search, where we check \(P(c_x, c_y)\) for each of the 864 center points and find the best one. We consider both orbits A and B with the following parameters: \(r_x = 45\) miles, \(r_y = 35\) miles, \(T = 1908\) seconds, \(s = 488\) MPH and \(h = 44,000\) feet for orbit A and \(r_x = 22.5\) miles, \(r_y = 17.5\) miles, \(T = 776\) seconds, \(s = 300\) MPH and \(h = 44,000\) feet for orbit B. We find that for orbit A, the best
We adopt a gradient-based search approach, which has lower complexity than exhaustive search. Results are provided for throughput optimization only for brevity and the same approach can be applied to duty cycle optimization. The best center is expected to be close to the centroid of the ground user positions that is used as the starting point. We then check its four neighbor grid points. To compare the gradient search results with the exhaustive search results, we consider the same grid size of 4 miles.\(^1\) We check four neighbors at \((c_x - 4, c_y - 4)\) miles, \((c_x - 4, c_y + 4)\) miles, \((c_x + 4, c_y - 4)\) miles, and \((c_x + 4, c_y + 4)\) miles. We then have two cases.

- **The performance by one of the four neighbors is better.** Suppose \((x, y)\) miles has the best performance \(P(x, y)\) among four neighbors. In this case, we move the current point from \((c_x, c_y)\) miles to \((x, y)\) miles. Then we further check neighbors of the new point in the next iteration.

- **The performance by the current point is better.** In this case, we claim that the current point \((c_x, c_y)\) miles is the best center and terminate the gradient search process.

For orbit A, we find the same best center \((104, -47)\) miles as the exhaustive search in three iterations (10 orbits evaluated). The search process is shown in Fig. 6. We first check the centroid location at \((108, -43)\) miles. In the first iteration, we find a better orbit \((108, -43)\) miles. In the second iteration, we again find a better orbit \((104, -43)\) miles. In the third iteration, we cannot find a better orbit and terminate the search process. Note that the gradient search may not result in the global optimum in general because the optimization function is not necessarily convex.

For orbit B, we find the same best center \((108, -43)\) miles as the exhaustive search in one iteration (5 orbits are evaluated). We can see that the gradient search can significantly decrease complexity (10 or 5 vs. 864) and find the same best center as the exhaustive search.

D. **Exhaustive Search for Orbit Radius**

Next, we select the best radius of the elliptical orbit. We fix center at \((104, -47)\) miles. Using the radius of orbit A as the boundary \(r_x = 45\) miles and \(r_y = 35\) miles, we search for radius by scaling 45 miles in X-direction and 35 miles in Y-direction. We vary the common scaling factor \(\beta\) in \([0.2, 5]\) (different scaling factors in X and Y directions could be used at the expense of higher complexity) and check the performance as a function of \(\beta\). The first approach is exhaustive search, where we check \([0.2, 5]\) with step size 0.2 (we check 25 scaling factors) and find the best scaling factor as 0.6, i.e., the best radius in 27 miles in X-direction and 21 miles in Y-direction. Exhaustive search has high complexity.

**E. Gradient Search for Orbit Radius**

We further consider a gradient search. We start with search space \([LB, UB]\) = \([0.2, 5]\) for scaling factor and a middle point \(m = 1\). We check two scaling factors \((1 + 0.2)/2 = 0.6\) and \((1 + 5)/2 = 3\). We then have three cases.

- **The performance \(P(0.6)\) is the best.** In this case, we change the search space as \([LB, UB]\) = \([0.2, 1]\) and the middle point as \(m = 0.6\).

- **The performance \(P(1)\) is the best.** In this case, we terminate and claim 1 is the best.
For the first case and the third case, we will further compare the throughput achieved by the middle point as \( m = 3 \).

For the second case, we will further compare the throughput achieved by the middle point with two other points in the next iteration.

A formal description of gradient search is as follows.

**Step 1:** Introduce smooth turning (green orbit) using turn radius 

\[
r = \frac{s^2}{(g \tan \theta)}.
\]

**Step 2:** Optimize the number of turning points.

In the second step, we use the following relationship among radius \( r \), speed \( s \), bank angle \( \theta \), gravity \( g \) and orbit period \( T \):

\[
r = \frac{s^2}{(g \tan \theta)}.
\]

**Step 3:** Optimize the number of turning points.

In the third step, force-level planning aims to minimize the average fuel consumption by optimally selecting the number of turning points, \( n_T \), such that the difference between polygon and elliptical curve lengths (or areas), denoted by \( d(n_T, t_S, t_T) \), is no more than a threshold, \( \tau \) (this constraint is imposed to bound the deviation from the optimal network performance) where the time spent on straight segment is \( t_S \) and the time spent on turn segment is \( t_T \). The optimization problem in this third step is written as

\[
\begin{align*}
\text{minimize} & \quad f_S \times t_S + f_T \times t_T \\
\text{subject to} & \quad d(n_T, t_S, t_T) < \tau \\
\text{variable} & \quad n_T.
\end{align*}
\]

We consider BD700 equivalent commercial aircraft at altitude \( h = 44,000 \) ft, speed \( s = 480 \) Mph, bank angle \( \theta = 20 \) degree, \( F_1 = 10.2 \) kg/min, \( F_2 = 14.0 \) kg/min (obtained from Base of Aircraft Data (BADA) [12]), turn radius \( r = 8.05 \) miles, and threshold \( \tau = 0.1 \). The best \( n_T \) is 6 when orbit A is considered with exhaustive search to optimize the throughput. The resulting operational orbit is shown in Fig. 8.

**V. FORCE-LEVEL PLANNING SOFTWARE**

We developed a *force-level planning software* with an interactive GUI in MATLAB. A snapshot of the force-level planning software’s GUI is shown in Fig. 9. The *Orbit* dialog box in Fig. 9 is used to set the orbit parameters. The orbit period is computed based on the total distance travelled and
the speed. Positions of subscribers are displayed (they can be configured for other scenarios). The Terrain Map shows the terrain profile as a heat map. The X-direction is East and Y-direction is North. The elliptical optimal orbit (magenta) and the operational orbit (brown) are overlaid. The 15 ground stations are also marked on the terrain (white ‘x’). The aircraft type is BD 700 and the bank angle is 20 degree.

The Optimization Results tab in Fig. 9 shows the best selected center for the orbit (similarly, the best radius can be added). The average duty cycle and the average throughput are computed based on the optimized center of the orbit across all subscribers. The throughput, the terrain elevation and the orbit elevation profiles are shown for any specified subscriber. The Throughput tab (top) shows the throughput in Kbps for five revolutions of the aircraft. The Elevation Profile tab (bottom left) shows the terrain elevation around the selected subscriber. The orbit Azimuth/Elevation, Az/El, tab (bottom right) shows the orbit elevation and azimuth from the subscriber’s perspective. The Azimuth is between 0 (East) and 360 degrees and elevation is between 0 and 90 degrees.

VI. CONCLUSION

In this paper, we presented a force-level planning approach that optimally selects and designs the best orbit of an airborne relay to maximize the aggregate performance of communication subscribers. The performance measures the duty cycle in terms of the signal-to-noise-ratio or the total throughput achieved by communication subscribers subject to realistic channel, terrain, antenna and aircraft characteristics for ground-to-air and air-to-ground communications. We presented different methods to generate an airborne relay orbit and then specialized results to elliptical orbits. In this approach, we selected the best center or radius (in X and Y-dimensions) as elliptical orbit parameters subject to the specified altitude and the speed of the aircraft. We performed the optimization via either exhaustive search or gradient search with reduced complexity. We provided the systematic mechanism to convert this elliptical orbit to an operational orbit by optimally selecting orbit waypoints and locating smooth turning points while minimizing the fuel consumption of the airborne relay. We designed and implemented the software for force-level planning that allows the planner to select different properties of airborne relay and communication subscribers, and returns the optimal selection of the airborne relay orbit.

REFERENCES

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