Abstract—This paper presents a network-centric mission planning system for airborne relay communications. Examples of airborne relays include aircrafts, such as the battlefield airborne communication node; unmanned aerial vehicles, such as Facebook’s Aquila; balloons, such as Google Project Loon; and nano/microsatellites. There are two integrated components of mission planning: 1) the subscriber planner (run on each subscriber’s radio, phone, or attached computer) that finds the optimal transmission scheme for each subscriber communicating to/through an airborne relay by accounting for realistic channel, traffic, and energy effects and 2) the orbit (force-level) planner (run at a centralized controller prior to deployment of the airborne relay) that optimally designs the orbit of an airborne relay to maximize the overall network performance. For a given orbit of an airborne relay, the subscriber planner combines channel measurement and prediction (by accounting for terrain, aircraft, and antenna effects) to calculate signal-to-noise-ratio (SNR) and link rate. Then, the subscriber planner decides when and at what rate to transmit while optimizing the expected throughput for uplink/relay communications. The subscriber planner is shown to significantly outperform random and SNR threshold-based transmissions. On the other hand, the orbit planner first selects the optimal parameters of an elliptical orbit to maximize the network performance measured by the subscriber planner. This elliptical orbit is converted to an operational orbit by optimally selecting waypoints and locating smooth turning points while minimizing the airborne relay’s fuel consumption. The orbit planner software with an interactive graphical user interface is implemented to return the optimal orbit for the airborne relay that optimizes the network performance measured by the subscriber planner.

Index Terms—Airborne relay, communications, duty cycle, mission planning, networking, optimization, orbit, orbit planning, throughput, trajectory, transmission scheduling.

I. INTRODUCTION

Airborne relays (or gateways) are used to provide mission-critical services such as connectivity, coverage, range extension, link bridging, and situational awareness to communication subscribers (ground, air, or maritime fixed or mobile users) [1]–[10]. One example of the airborne relay that aims to serve communication subscribers is the battlefield airborne communication node (BACN) [11]. Other examples include unmanned aerial vehicles (UAVs), such as Facebook’s Aquila [12], Google’s Balloon (Project Loon [13]), and nano/microsatellites that aim to provide communications subscribers with connectivity (e.g., Internet) in remote areas.

From the network communications perspective, the overall performance depends on: i) the transmission decisions of each communication subscriber (for a given orbit or trajectory of the airborne relay) to optimize its performance (such as throughput) [1] and ii) the selection of the airborne orbit to provide subscribers with the best network communications support [2]. Therefore, network-centric mission planning system with airborne relays has two integrated components: 1) the subscriber planner (at the individual user level) and 2) the orbit (force-level) planner (at the network level).

1) Subscriber planner: For a given orbit $O$, the subscriber planner optimizes the performance $P_i(O)$ of each subscriber $i$ communicating to/through the dedicated airborne communication platform and schedules when and at what rate to transmit as the airborne relay moves. The objective is to maximize the network-centric performance $P_i(O)$ (e.g., throughput) by taking into account channel, traffic, and energy effects. The subscriber planner is run on each subscriber’s radio, phone, or attached compute.

2) Orbit planner: For a given set of communication subscribers $S$, the orbit planner searches for the best orbit $O_S$ by computing and comparing the total network performance $\sum_{i \in S} P_i(O)$ achieved by all subscribers for each orbit $O$ (note that $P_i(O)$ is measured by the subscriber planner that is run for subscriber $i$). This performance is optimized by selecting the best orbit parameters of the airborne relay subject to different orbit constraints and communication requirements. The orbit planner is run at a centralized controller prior to deployment of the relay.

A. Subscriber Planner

The subscriber planner determines the optimal transmission decisions for subscribers communicating to/through a dedicated airborne relay. For a given orbit/trajectory of an airborne relay, channels between the relay and subscribers are predicted offline (using advanced channel generation tools such as [14]) and used at the communication subscriber or at the airborne relay.
together with channel estimates obtained in real time from the relay’s beacon/heartbeat signals. We consider various communication patterns such as uplink communications (from a subscriber to the airborne relay) and relay communications (first from a subscriber to the relay and then from the relay to another subscriber). Each subscriber (or airborne relay for downlink) individually determines when and at what rate to transmit by considering current and future channel variations (predicted offline and estimated in real time from the received signals), data traffic, and energy constraints (assuming battery-operated radios at subscribers). The steps of the subscriber planner are:

1) For realistic channel modeling and prediction, the CBAR (Channel modeling tool based on bidirectional analytic ray tracing and radiative transfer) [14] is used to reliably predict the multipath time-varying channel. The CBAR considers real terrain information, radiation patterns of transmitter and receiver antennas, position, velocity and orientation of transmitter, and signaling bandwidth to compute time-varying multipath channel. This offline prediction is combined with real-time channel estimation by subscribers that receive the beacon/heartbeat signals.

2) Signal-to-noise-ratio (SNR) is computed first and then converted to link rate by using waveform specifications. As an example, Link 16 communication specifications [15] are used to compute link rates.

3) The subscriber planner with the optimal transmission scheme is designed to make transmission decisions for both uplink and relay cases by considering channel quality, traffic load, and energy consumption to maximize the expected throughput. The downlink case is designed similar to the uplink case.

Two other schemes are designed for comparison purposes such that whenever data and energy are available: 1) the threshold-based scheme decides to transmit if SNR exceeds some threshold and 2) the benchmark scheme makes transmissions randomly. We compare the optimal transmission scheme with the threshold-based scheme and the benchmark scheme for both uplink and relay cases. We show that the optimal scheme achieves 60% more throughput compared to the benchmark scheme for any SNR, traffic load, and energy budget. On the other hand, the threshold-based scheme strongly relies on the selection of the SNR threshold in advance. The optimal scheme does not depend on a parameter selection in advance and provides steady throughput with fewer fluctuations over time (this is an important feature for multimedia applications such as video).

B. Orbit Planner

A typical measure for the optimal orbit selection is the coverage (or duty-cycle) such as the percentage of time a communication subscriber is connected with (i.e., it is within the transmission range of) the airborne relay [3], [8]. This measure does not fully consider the underlying communication characteristics. We consider maximizing either the duty cycle that reflects the percentage of time the SNR exceeds a given threshold, or more realistically the actual network throughput, namely, the number of packets delivered between communication subscribers and the airborne relay per unit time. Duty cycle accounts for the following: i) predicted channels (calculated based on terrain profile, distance to airborne relay, and antenna characteristics [14]) and ii) estimated channels (from pilot signals received from the airborne relay). In addition to those factors, the throughput also accounts for: iii) packet traffic; iv) battery energy properties; and v) interference effects, among other radio-related factors.

The choice for orbit parameters depends on the method that is used for orbit generation. Examples of orbit parameters include the center, the radius (both in X- and Y-directions), and the orientation (defined as rotation with respect to X- and Y-axis) of elliptical orbits (see Fig. 1) or the waypoints for general orbits in two dimensions (2-D). The orbit planner starts with the parameter optimization for an elliptical orbit and then converts it to an operational orbit as follows.

1) We optimally select the center, the radius (in semimajor and semiminor axis or X- and Y-direction), and the orientation of an elliptical orbit (which is an approximation of real orbits). We select these orbit parameters to optimize the duty cycle or the network throughput subject to the specified aircraft properties (altitude and speed). We consider both exhaustive search and gradient search with reduced complexity to solve the optimization problem.

2) We modify this elliptical orbit to an operational orbit by selecting the best set of waypoints on this orbit and locating smooth turning points. We select the number of waypoints to minimize the fuel consumption of the airborne relay by limiting the number of turning points, while constraining the deviation from the elliptical orbit and introducing smooth turns depending on the speed and bank angle of the aircraft. We use real rates of aircraft fuel consumption from base of aircraft data (BADA) files [16].

We wrap this system in the orbit planner software implementation that allows the planner to select the relay and subscriber specifications and returns the optimal orbit of the airborne relay.

C. Summary of Contributions and Paper Organization

Our contributions are summarized below as follows.

1) Realistic channel modeling/prediction and link rate computation based on terrain and aircraft characteristics.

2) Optimization of transmission decisions based on channel, traffic, and energy conditions.

3) Systematic use of high-fidelity network communication measures to select the optimal orbit parameters of an airborne relay by exhaustive and gradient search methods.
4) Optimization of fuel consumption and turning point selection to establish operational orbits for airborne relays.

The rest of the paper is organized as follows. Section II introduces the problem of orbit generation for airborne relays. Section III presents channel modeling/prediction and link rate computation. Section IV derives the optimal transmission scheme of the subscriber planner and evaluates its performance. Section V presents the optimal selection of elliptical orbit parameters through exhaustive and gradient-search approaches. Section VI presents the conversion of an elliptical orbit and describes the implementation of the orbit planner software. Section VII concludes the paper.

II. ORBIT GENERATION METHODS

We discuss different ways to generate the orbit for an airborne relay and identify the parameters to be selected.

A. Elliptical Orbit Generation

The X and Y coordinates of the elliptical orbit are

\[ x(t) = c_x + r_x \cos \left( \frac{2\pi t}{T} \right) \cos \theta_R - r_y \sin \left( \frac{2\pi t}{T} \right) \sin \theta_R \]

\[ y(t) = c_y + r_x \cos \left( \frac{2\pi t}{T} \right) \sin \theta_R + r_y \sin \left( \frac{2\pi t}{T} \right) \cos \theta_R \]

where \((c_x, c_y)\) is the center of orbit, \(r_x\) and \(r_y\) are radii along X- and Y-directions, respectively, \(\theta_R\) is orientation in counterclockwise direction, as shown in Fig. 1, and \(T\) is the orbit time. Different parameters \(c_x, c_y, r_x, r_y, \theta_R\), and \(T\) (or speed \(s\)) can be varied to generate different orbits. One sample orbit A (the larger one in red) and one sample orbit B (the smaller one in black) are shown in Fig. 2, where \(r_x = 45 \text{ mi}\), \(r_y = 35 \text{ mi}\), \(T = 1908 \text{ s}\), \(s = 488 \text{ mi/h (miles per hour)}\), and \(h = 44 000 \text{ ft}\) for orbit A, and \(r_x = 22.5 \text{ mi}\), \(r_y = 17.5 \text{ mi}\), \(T = 776 \text{ s}\), \(s = 300 \text{ mi/h}\), and \(h = 44 000 \text{ ft}\) for orbit B. In this paper, we vary parameters \(c_x, c_y, r_x, r_y, \text{ and } \theta_R\), and keep others fixed to generate different orbits.

B. Waypoint-Based Orbit Generation

Another way to generate an airborne relay orbit is to specify its waypoints. For example, the CBAR tool [14] can be used to generate an airborne relay orbit. We first add waypoints using the CBAR user interface. Based on waypoints, speed, and altitude information, the CBAR generates the route using AGI’s route design library from Systems Tool Kit [17] integrated within the CBAR (illustrated in Fig. 3). The CBAR uses real terrain data and real aircraft models in channel computation and route generation. An example of waypoints generated by the CBAR is shown in Fig. 4.

Next, we consider the optimal selection of orbit parameters. We start with optimizing the elliptical orbit parameters. Then, we introduce waypoints to improve the orbit.

III. CHANNEL MODELING/PREDICTION AND LINK RATE COMPUTATION

Assume an airborne relay orbit (e.g., BACN [11]) is generated by one of the methods mentioned in Section II. For a given orbit, we use the CBAR [14] to predict channel qualities with respect to each subscriber and the airborne relay based on terrain profile. By applying physical optics and ray tracing techniques, the CBAR computes the time-varying multipath channel response by using terrain information, transmitter and receiver antenna patterns, and velocity and orientation of transmitter and bandwidth. This predicted channel information is stored at the subscriber and the airborne relay. Alternative channel modeling methods include parametric path loss models, terrain integrated...
rough earth model (TIREM) [18], and irregular terrain model (ITM)[19], where higher order effects (e.g., out-of-plane terrain scattering, resolvable multipath) are typically not accounted for.

Each subscriber has two SNRs, SNR_{CBAR} (predicted by the CBAR) and SNR_{est} (estimated from beacon/heartbeat signals transmitted by the airborne relay), and two corresponding link rates are computed, R_{CBAR} and R_{est}. In case of simulations, SNR_{est} is generated by adding a random error term to SNR_{CBAR} and converted to R_{est}. A communication subscriber has the estimated rate R_{est} for current time only. For future time instants, it uses the rate R_{CBAR} predicted by CBAR. These two rates are combined to the rate estimate r(t) for any time t. Next, we discuss how to obtain SNR_{CBAR} and R_{CBAR}.

A. SNR Computation

The CBAR generates channel model of aerial communications by combining terrain, aircraft position, orientation and velocity, and antenna pattern of aircraft and subscribers. To generate the heat map of SNR and throughput over the entire region (as the airborne relay moves), we use only limited features of the CBAR with fast computation to check for terrain blockage at each location and generate propagation path loss with free-space propagation model. For the scenario discussed in Section V, the CBAR estimates 150 channels for a sample scenario of 15 ground nodes and 10 airborne relay positions on an orbit. We assume transmitter power of 30 dBm, bandwidth of 5 MHz, and receiver noise figure of 4 dB (used in the SNR computation) for channel computation.

For any pair of subscriber and airborne relay positions at any time instant, SNR_{CBAR} is computed as follows. First, the path loss between transmitter and receiver is computed using the CBAR, and then the SNR and the link rate are computed according to analytical computations based on Link 16 waveform specifications [15]. Note that Link 16 uses time-division multiple access (TDMA) to avoid interference among subscribers. We express the SNR in dB as follows:

\[
\text{SNR} = \text{SNR}_{\text{CBAR}} - L - 10 \log_{10}(k T_s) - F - 10 \log_{10}(B)
\]

\[
= 30L + 174 - 4 - 67
\]

\[
= 133 - L
\]

where \(P_t\) is transmitter power in dBm, \(L\) is pathloss in dB, \(k\) is Boltzmann constant \(1.38 \times 10^{-23} \text{ J/K}\), \(T_s\) is standard temperature 290 K, \(F\) is receiver noise figure in dB, and \(B\) is bandwidth in Hz.

Fig. 2 shows one example of terrain profile (with examples of subscriber locations marked by “x”). 2-D and 3-D representations of the SNR between the airborne relay position at (145, −5) measured in miles and all possible subscriber positions are shown in Fig. 5, where each pixel refers to the X and Y coordinates for a potential subscriber’s location, red means high SNR, blue means low SNR, and white means no line of sight. Note that we omit the Z coordinate for a subscriber since it is uniquely determined by the X and Y coordinates of the location for any terrain profile and the Z-axis of the figure is allocated to indicate the SNR value.

B. Link Rate Computation

We convert SNR or pathloss to link rate estimate using transmitter/receiver specifications (by either analytical model or radio measurement). Below is an example using the known features of Link 16 waveform, as reported in [15]. Link 16 uses 32 chip sequence to represent 5-b symbols. The ON duration of a single pulse is 6.4 \(\mu\)s and the OFF duration is 6.6 \(\mu\)s, resulting in pulse duration of 13 \(\mu\)s. Therefore 5 bits are transmitted in a single-pulse duration. We neglect the overhead associated with parity and any error correction. The peak data rate is \(R_p = 5/13 \text{ ms} = 384.6 \text{ kb/s}\). We approximate the symbol error probability as \(\text{SER} = \min \{ 31Q(\sqrt{32 \times \text{SNR}}), 31/32 \} \), where \(Q\)-function is the tail probability of the standard normal distribution. The first term in the minimum corresponds to the pairwise error probability of 32-ary orthogonal modulation, and the second term limits the maximum symbol error probability to 31/32.

The resulting instantaneous link rate is given by

\[
\text{PER} = 1 - (1 - \text{SER})^{8N/5}
\]

The resulting instantaneous link rate is given by

\[
R = R_p \times (1 - \text{PER}) \text{ b/s}
\]

We convert SNR values to rate values for the airborne relay position at (145, −5). The two and three-dimensional representations of link rate at one time instant are shown in Fig. 6, where each pixel denotes a subscriber position and red and blue pixels mean high and low link rates, respectively. We focus on uncoded transmissions, and it is easy to extend the results to account for channel coding. The bit error and symbol error rate results un-
under different channel, and interference conditions can be found in [20].

IV. OPTIMAL TRANSMISSION DECISIONS BY SUBSCRIBER PLANNER

For a given orbit $O$ (from Section II) and given channel prediction results (from Section III), the subscriber planner is run at the radio of each subscriber $i$ and airborne relay, and the subscriber planner makes a transmission decision on when (which slot) and how much data (number of bits per slot) to transmit from a subscriber to the airborne relay (in the uplink case and in the first hop of the relay case) or from the airborne relay to a subscriber (the second hop of the relay case). Adaptive transmission scheduling corresponds to medium access control [21], [22] in radio communications and provides each radio with cognitive radio capability [23] to learn and adapt to spectrum dynamics. The uplink and relay cases are illustrated in Fig. 7. The downlink case (from the relay to the subscriber) can be designed similar to the uplink case, and therefore it is omitted here. The objective of a subscriber $i$ is to optimize its performance $P_i(O)$ that is measured by its expected throughput (which can be extended to connectivity and other measures).

We consider a fixed transmission rate without feedback or retransmissions. A packet is removed from the queue once it is transmitted. The decision to transmit or not at any time $t$ is based on: 1) current and future link rates, 2) queue length and estimated packet arrival rate, and 3) energy budget to spend in one round (when energy efficiency is a concern, e.g., for battery-operated radios). The packet arrival rate $r_{arr}$ at time $t$ is estimated by the average arrival rate until time $t$.

A. Uplink Case

When a subscriber makes a transmission decision at any given time, it is possible that the subscriber postpones transmissions (as illustrated in Fig. 8) because if it does not postpone, it will not have enough data or energy to transmit when better channels arrive. We formulate the subscriber planner as an optimization problem. We define $Q(t)$ as the queue length at time $t$, $R$ as the transmission rate per time slot, $r(t)$ as the estimated link rate at time $t$, $E$ as the energy budget, and $P$ as the energy consumption per TDMA slot. Then, $R_{\text{max}}(t) = \min\{R, Q(t), \frac{E(t)}{R}\}$ is the maximum data that a subscriber can transmit at time $t$. For the optimal transmission, we consider the following cases.

1) If a subscriber transmits, then
   a) the expected throughput is $r(t)$, $Q(t)$ is updated as $Q(t) - R$, and the remaining budget $E(t)$ is updated as $E(t) - P$, when $Q(t) \geq R$ and $E(t) \geq P$;
   b) the expected throughput is $\frac{E(t)}{R}r(t)$, $Q(t)$ is updated as $Q(t) - \frac{E(t)}{R}R$, and the remaining budget $E(t)$ is updated as zero, when $Q(t) \geq R$ and $E(t) < P$;
   c) the expected throughput is $\frac{Q(t)}{R}r(t)$, $Q(t)$ is updated as zero, and the remaining budget $E(t)$ is updated as $E(t) - \frac{Q(t)}{R}P$, when $Q(t) < R$ and $E(t) \geq \frac{Q(t)}{R}P$;
   d) the expected throughput is $\frac{E(t)}{R}r(t)$, $Q(t)$ is updated as $Q(t) - \frac{E(t)}{R}R$, and the remaining budget $E(t)$ is updated as zero, when $Q(t) < R$ and $E(t) < \frac{Q(t)}{R}P$.

2) If a subscriber does not transmit, then the expected throughput is zero and $Q(t)$ and $E(t)$ are unchanged. These conditions reduce to the following two cases.
1) If $Q(t) \geq R$ always holds, then a subscriber should always transmit in the $\lfloor \frac{R}{P} \rfloor$ best time slots.
2) If transmitting data now makes $Q(\tau) < R$ or $E(\tau) < P$ in a future time slot with $r(\tau) > r(t)$, then reducing data transmitted now and increasing data transmitted at time $\tau$ can increase the total expected throughput.

Based on these two cases, we check whether transmitting data of size $R_{\text{max}}(t)$ (in bits) now makes $Q(\tau) < R$ or $E(\tau) < P$ in a future time slot with $r(\tau) > r(t)$. If no, this subscriber transmits data of size $R_{\text{max}}(t)$ now. Otherwise, it should transmit data less than $R_{\text{max}}(t)$ (not transmitting is an extreme case). This check is based on assumptions that: i) data of size $r_{\text{arr}}$ arrive at the beginning of each time slot ($r_{\text{arr}}$ at time $t$ is estimated by time-averaging arrivals until time $t$); ii) data of size $R_{\text{max}}(t)$ are transmitted now; iii) data are not transmitted in any future time slots with $r(\tau) \leq r(t)$; and iv) data of size $R$ are transmitted in any future time slots with $r(\tau) > r(t)$ unless $Q(\tau) < R$ or $E(\tau) < P$. We limit this check for the current covered period. This analysis leads to the following optimal transmission decision:

- **Optimal subscriber planner:** At time $t$, a subscriber transmits the data $D(t) \leq R_{\text{max}}(t)$ of the largest size that makes $Q(\tau) \geq R$ and $E(\tau) \geq P$ in future time slots $\tau$ with $r(\tau) > r(t)$.

In addition, we consider two simpler but suboptimal schemes for comparison purposes:

- **Threshold-based scheme:** At time $t$, a subscriber transmits, whenever SNR exceeds some threshold $\gamma$ and data and energy are available.
- **Benchmark scheme:** At time $t$, a subscriber transmits with some probability, when data and energy are available.

To be fair, we set the same energy budget $E$ per orbit time for all these schemes. Given energy consumption is $P$ per slot and there are ten (TDMA) slots per orbit time, the transmission probability is $E/(10P)$ for each slot in the benchmark scheme. Thus, as long as there is remaining energy budget and there are buffered packets, the benchmark scheme decides to transmit with probability $E/(10P)$. Once a node transmits, the achieved rate is $R_{\text{est}}$, which is estimated from pilot signals of the airborne relay.

### B. Relay Case

In the relay case, the first hop is an uplink transmission and the second hop is a downlink transmission. We can apply a similar idea from uplink to downlink case by considering the following differences. Traffic to uplink is given as an input while traffic to downlink is the output of the first hop and thus depends on transmission schemes. In addition, throughput is measured in the second hop, whereas the energy consumption is measured in both hops.

### C. Performance Evaluation of Subscriber Planner

We use 15 subscribers (marked as white “x”) and two orbits of the airborne relay, as shown in Fig. 9, where orbit 1 (marked in magenta) is generated by an elliptic orbit formula, and orbit 2 (marked in yellow) is generated by the CBAR using waypoints.
put; 2) it applies to any choice of traffic load and energy budget; 3) it provides steady throughput over time with fewer fluctuations (important for multimedia); and 4) it does not need a “good” SNR threshold computed in advance.

Table II shows the average performance (averaged over ground nodes and time) and minimum performance (averaged over time for the worst ground node) of throughput and duty cycle (percentage of time SNR > 1.8 dB) for two orbits. Orbit 2 is better in terms of throughput, and orbit 1 is better in terms of duty cycle. Duty cycle is limited to SNR comparison with a threshold, whereas the optimal scheme sustains the throughput optimality independent of SNR.

3) Relay Case: We now consider the relay case, where a subscriber (source) transmits its data to the airborne relay that forwards this data to another subscriber (destination). Instantaneous and average throughput values are shown in Figs. 14 and 15, respectively, for source node 7 at location (128, −22) and destination node 13 at location (164, −90). The optimal scheme outperforms the other two schemes significantly.

### Table I

<table>
<thead>
<tr>
<th>Transmission Scheme</th>
<th>Orbit 1</th>
<th>Orbit 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal</td>
<td>14.487 kb/s</td>
<td>17.759 kb/s</td>
</tr>
<tr>
<td>Threshold-based</td>
<td>13.308 kb/s</td>
<td>16.575 kb/s</td>
</tr>
<tr>
<td>Benchmark</td>
<td>8.737 kb/s</td>
<td>9.612 kb/s</td>
</tr>
</tbody>
</table>

### Table II

<table>
<thead>
<tr>
<th></th>
<th>Average Throughput</th>
<th>Minimum Throughput</th>
<th>Average Duty Cycle</th>
<th>Minimum Duty Cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orbit 1</td>
<td>14.97 kb/s</td>
<td>13.59 kb/s</td>
<td>0.91</td>
<td>0.20</td>
</tr>
<tr>
<td>Orbit 2</td>
<td>16.91 kb/s</td>
<td>14.34 kb/s</td>
<td>0.71</td>
<td>0.30</td>
</tr>
</tbody>
</table>
and the threshold-based scheme (with an appropriate threshold $\gamma = 4.7$ dB) performs much better than the benchmark scheme.

V. OPTIMAL SELECTION OF AN ELLIPTICAL ORBIT

The orbit planner compares the network communications performance under different orbits and systematically selects the best orbit parameters. The orbit planner has two steps.

1) Find the best elliptical orbit to maximize the total performance (throughput or duty cycle) of communication subscribers. Throughput maximization uses the subscriber throughput returned by each individual subscriber planner, while duty cycle maximization is independent of the subscriber planners.

2) Convert the elliptical orbit to waypoints and smoothen turning points while minimizing the fuel consumption.

In Section V, we address Step 1, where the goal is to find the center $(c_x, c_y)$, the best radius $(r_x, r_y)$ and/or the best orientation $\theta_R$ (shown in Fig. 1) of the best elliptical orbit $O^*$. We will address Step 2 in Section VI.

A. Network Performance for a Given Orbit

We developed the subscriber planner in Section V, where the performance of each subscriber is individually optimized for a given airborne relay orbit. This optimization accounts for channel variations (predicted offline and estimated in real time from relay’s pilot signals), data traffic, and energy consumption (subject to energy constraints for battery-operated units). Next, we flip the problem and compare different orbits in terms of the overall performance of communication subscribers. We will present results for two performance metrics: duty cycle (the percentage of time the SNR is greater than a radio-specific threshold) and throughput (the number of packets received by destination per unit time) sustained by the subscriber. Other metrics can be handled by similar optimization procedures. Consider 15 subscribers (marked as white “×”) and two orbits of the airborne relay, as shown in Fig. 9, where orbit 1 (marked in magenta) is generated by an elliptical orbit formula, and orbit 2 (marked in yellow) is generated by the CBAR using waypoints. As reported earlier, Table II shows the average values of throughput and duty cycle (percentage of time SNR $> 1.8$ dB) for two orbits shown in Fig. 16. Orbit 2 is better than orbit 1 in terms of throughput, and orbit 1 is better than orbit 2 in terms of duty cycle.

The next question is how to select the best orbit parameters (either the center or the radius) in a systematic way.
B. Search for Orbit Center

1) Exhaustive Search: We consider 15 ground users, as shown in Fig. 9. These users are located in the area of $[36, 176] \times [-90, 0]$. We search for center within the same area that has resolution (grid size) of 4 mi. That is, $c_x$ can be selected from 36,40,...,176, and $c_y$ can be selected from $-90, -86, \ldots, 2$. The search space has $36 \times 24 = 864$ points. The overall performance can be expressed as a function $P(c_x, c_y)$, which is determined by the subscriber planner.

The first approach is the exhaustive search, where we check $P(c_x, c_y)$ for each of the 864 centers and find the best one. We consider both orbits A and B with the following parameters: $r_x = 45$ mi, $r_y = 35$ mi, $T = 1908$ s, $s = 488$ mi/h, and $h = 44 000$ ft for orbit A, and $r_x = 22.5$ mi, $r_y = 17.5$ mi, $T = 776$ s, $s = 300$ mi/h, and $h = 44 000$ ft for orbit B. We find that for orbit A, the best center is $(104, -47)$ mi (red orbit in Fig. 16) for throughput maximization and $(172, -58)$ mi (yellow orbit in Fig. 16) for duty cycle maximization.

For orbit B, the best center is $(108, -43)$ mi (white orbit in Fig. 16) for throughput maximization and $(152, -58)$ mi (black orbit in Fig. 16) for duty cycle maximization. When we compare orbits A and B, we find that orbit B with the best center is better than orbit A with the best center in terms of both throughput and duty cycle. We will further address the search for the best radius in Section V-C and the search for the best orientation in Section V-D.

2) Gradient Search: We apply gradient-based search with lower complexity than exhaustive search. Results are provided for throughput optimization only for brevity, and the same approach can be applied to duty cycle optimization. The best center is expected to be close to the centroid of the ground user positions that is used as the starting point. We then check its four neighbor grid points. To compare the gradient search results with the exhaustive search results, we consider the same grid size of 4 mi.\footnote{We note that the gradient search can be improved, e.g., by adjusting step size or moving direction based on the potential performance improvement. We choose the current implementation such that we can easily compare its results with the exhaustive search results.} We check four neighbors at $(c_x - 4, c_y - 4)$ mi, $(c_x - 4, c_y + 4)$ mi, $(c_x + 4, c_y - 4)$ mi, and $(c_x + 4, c_y + 4)$ mi. We then have two cases.

1) The performance by one of the four neighbors is better. Suppose $(x, y)$ mi has the best performance $P(x, y)$ among four neighbors. In this case, we move the current test point from $(c_x, c_y)$ mi to $(x, y)$ mi. Then we further check neighbors of the new test point in the next iteration.

2) The performance by the current point is better. In this case, we claim that the current point of $(c_x, c_y)$ mi is the best center and terminate the gradient search process.

For orbit A, we find the same best center $(104, -47)$ mi as the exhaustive search in three iterations (10 orbits evaluated). The search process is shown in Fig. 17. We first check the centroid location at $(108, -43)$ mi (white orbit in Fig. 17). In the first iteration, we find a better orbit (yellow orbit in Fig. 17) centered at $(104, -43)$ mi. In the second iteration, we again find a better orbit (red orbit in Fig. 17) centered at $(104, -47)$ mi. In the third iteration, we cannot find a better orbit and terminate the search process. Note that the gradient search may not result in the global optimum in general because the optimization function is not necessarily convex. For orbit B, we find the same best center $(108, -43)$ mi as the exhaustive search in one iteration (5 orbits are evaluated). We can see that the gradient search can significantly decrease complexity (10 or 5 versus 864) and find the same best center as the exhaustive search.

C. Search for Orbit Radius

1) Exhaustive Search: Next, we select the best radius of the elliptical orbit. We fix center at $(104, -47)$ mi. Using the radius of orbit A as the boundary ($r_x = 45$ mi and $r_y = 35$ mi), we search for radius by scaling 45 mi in X-direction and 35 mi in Y-direction. We vary the common scaling factor $\beta$ in $[0, 2, 5]$ (different scaling factors in X- and Y-directions could be used at the expense of higher complexity) and check the performance as a function of $\beta$. The first approach is the exhaustive search, where we check the range $[0, 2, 5]$ with step size 0.2 (we check 25 scaling factors) and find the best scaling factor as 0.6, i.e., the best radius is 27 mi in X-direction and 21 mi in Y-direction. Exhaustive search has high complexity.

2) Gradient Search: We consider a gradient search with reduced complexity. We start with search space $[LB, UB] = [0, 2, 5]$ for scaling factor and a middle point $m = 1$. We check two scaling factors $(1 + 0.2)/2 = 0.6$ and $(1 + 5)/2 = 3$. We define $P(x_s)$ as the performance when $x_s$ is the scaling factor. We then have three cases in terms of $P(0.6), P(1)$, and $P(3)$.

1) $P(0.6)$ is the best. In this case, we change the search space as $[LB, UB]$ and the middle point as $m = 0.6$.

2) $P(1)$ is the best. In this case, we terminate and claim 1 is the best.

3) $P(3)$ is the best. In this case, we change the search space as $[LB, UB] = [1, 5]$ and the middle point as $m = 3$.

For the first case and the third case, we will further compare the middle point with two other points in the next iteration.

A formal description of gradient search is as follows.

1) The initial search space for scaling factor is $[LB, UB]$ and the middle point is $m = 1$, where $LB < m$ and $UB > m$.
2) If the maximum number of iterations is not reached, we compare the throughput achieved by the middle point with 
\[ r_1 = (LB + m)/2 \text{ and } r_2 = (UB + m)/2. \]
a) If \( P(r_1) \) is the best, we change the search space as \([LB, m]\) and the middle point as \( r_1 \).
b) If the performance \( P(m) \) is the best, we terminate and claim \( m \) is the best orbit size.
c) If the performance \( P(r_2) \) is the best, we change the search space as \([m, UB]\) and the middle point as \( r_2 \).

In gradient search, we have \( LB = 0.2, UB = 5, m = 1, r_1 = 0.6, \text{ and } r_2 = 3 \) in the first iteration. We check scaling factors \( m = 1, r_1 = 0.6, \text{ and } r_2 = 3 \). It turns out that scaling factor 0.6 achieves the largest throughput. In the second iteration, we have \( LB = 0.2, UB = 1, m = 0.6, r_1 = 0.4, \text{ and } r_2 = 0.8 \). We check scaling factors \( m = 0.6, r_1 = 0.4, \text{ and } r_2 = 0.8 \). It turns out that the scaling factor 0.6 still achieves the largest throughput. Thus, we terminate with the optimal scaling factor 0.6. We checked five scaling factors to find the same optimal scaling factor. Scaling factor 0.6 corresponds to radius \( r_x = 27 \text{ mi} \) and \( r_y = 21 \text{ mi} \) in X- and Y-directions, respectively.

D. Search for Orientation

Finally, we tune orbit parameter \( \theta_R \), i.e., the degree to rotate the orbit counter-clockwise, while fixing the orbit center as \((104, -47) \text{ mi}\) and the orbit radius as \( r_x = 27 \text{ mi} \) and \( r_y = 21 \text{ mi}\) (the optimal values that are identified in previous sections). Note that we obtain the same orbit if we rotate an orbit by \( \theta \) and \( \theta + \pi \) degree. Thus, the search space to tune \( \theta_R \) is \([0, \pi]\). In particular, we check \( \theta_R \) at \( K \) values \( 0, \frac{\pi}{K}, \ldots, \frac{(K-1)\pi}{K} \), and find the value that achieves the best performance. For \( K = 10 \), we find that \( \theta_R = \frac{2\pi}{K} \) achieves the best performance.

VI. CONVERSION TO AN OPERATIONAL ORBIT

From the operational point of view, it may be desirable to minimize the number of turns. This translates to \textit{fuel efficiency}, as turning consumes more fuel than flying straight. Denote \( f_S \) and \( f_T \) as the fuel consumption for straight and turning segments, respectively, per unit time. An elliptical orbit may not be preferable because the relay is always turning and \( f_T > f_S \). To make an operational orbit, we pursue the following two objectives.

1) New orbit should be similar to the elliptical orbit to constrain the change on the optimized performance.
2) New orbit should have lower fuel consumption.

We follow the following three steps (see Fig. 18) to make an operational orbit.

1) Fit a convex polygon (red orbit) to the elliptical orbit (blue orbit).
2) Introduce smooth turning (green orbit) by using the turn radius that depends on speed and bank angle.
3) Optimize the number of turning points.

We developed the \textit{orbit planner software} with an interactive GUI in MATLAB. A snapshot of this GUI is shown in Fig. 19. The \textit{Orbit} dialog box in Fig. 19 is used to set the orbit parameters. The orbit period is computed based on the total distance travelled and the speed. Positions of subscribers are displayed (they can be configured for other scenarios). The \textit{Terrain Map} shows the terrain profile as a heat map. The X-direction is east and Y-direction is north. The elliptical optimal orbit (marked in magenta) and the operational orbit (marked in brown) are overlaid. The 15 ground stations are also marked on the terrain (marked as white “+”). The aircraft type is BD-700, and the bank angle is 20°.

The \textit{Optimization Results} tab in Fig. 19 shows the best selected center for the orbit (similarly, the best radius can be added). The average duty cycle and the average throughput are computed based on the optimized center of the orbit across all subscribers. The throughput, the terrain elevation, and the orbit elevation profiles are shown for any specified subscriber. The \textit{Throughput} tab (top) shows the throughput in kilobits per second for five orbit turns of the aircraft. The \textit{Elevation Profile} tab (bottom left) shows the terrain elevation around the selected sub-
sider. The orbit Azimuth/Elevation, Az/El, tab (bottom right) shows the orbit elevation and azimuth from the subscriber’s perspective. The azimuth is between 0° (East) and 360°, and elevation is between 0° and 90°.

We consider BD-700 equivalent commercial aircraft at altitude \( h = 44,000 \text{ ft} \), speed \( s = 480 \text{ mi/h} \), bank angle \( \phi = 20° \), \( F_1 = 10.2 \text{ kg/min} \), \( F_2 = 14.0 \text{ kg/min} \) (obtained from BADA[16]), turn radius \( \sigma = 8.05 \text{ mi} \), and threshold (on the deviation of length) \( \delta = 0.1 \). The best \( n_T \) is 6 when orbit A is considered with exhaustive search to optimize the throughput. The resulting orbit is shown on top of terrain map in Fig. 19.

VII. CONCLUSION

We presented a mission planning system with two integrated components to support subscriber communications with an airborne relay:

1) **Subscriber planner** optimizes transmissions (when and at what rate) for each subscriber communicating to/through an airborne relay by accounting for channel (predicted offline or estimated in real time), traffic, and energy conditions to maximize the duty cycle or the expected throughput. Using real terrain, aircraft, and waveform characteristics, the subscriber planner is shown to achieve significant (uplink and relay) throughput gains compared to SNR threshold-based and benchmark (random transmission) schemes.

2) **Orbit planner** optimally selects the orbit of an airborne relay to maximize the aggregate performance of communication subscribers (such as the throughput returned by the subscriber planner). First, an elliptical orbit is generated by optimally selecting the best center, radius (in X- and Y-directions), and/or orientation through exhaustive or gradient search. Then, this elliptical orbit is converted to an operational orbit by optimally selecting waypoints and locating smooth turning points to minimize the fuel consumption.

REFERENCES


Authors’ photographs and biographies not available at the time of publication.